Drums Along the Efficient Frontier

Investment performance of individual properties depends less on property type and location than generally believed.

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Investment performance of individual properties depends less on property type and location than generally believed.

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Michael S. Young and D. Wylie Greig

The real estate industry tends to describe property investments in terms of their physical characteristics—that is, their type and location. It is only natural, then, to assume that investment performance will follow the same groupings. Investors expect that properties with similar physical characteristics and similar business operations will exhibit similar patterns of returns. As a result, owners, investment managers, appraisers, consultants, and industry organizations record and report property returns by property type and geographic location.1

When the pension real estate industry began to focus on portfolio management, asset allocation, and diversification issues during the late 1980's, it was a foregone conclusion that investment performance would be analyzed by property type and geographic location. After all, that was the way the data were recorded; that was the way real estate people had been doing it for decades. Researchers who looked to the securities industry for analytical tools found that these classifications of real estate resembled securities industry classifications. For example, stocks are classified according to industry groups: automotive, pharmaceutical, energy, and so on. Furthermore, when measured in the aggregate, real estate returns for each property type looked different from those for the other property types. Returns reported for properties in one region also tended to differ from those in other regions. These observations seemed to validate the idea that property type and location could be used to distinguish performance among physically and locationally different segments of the real estate asset class.

Applying Modern Portfolio Theory to Real Estate

The observations also suggested that an opportunity existed to use well-established statistical techniques drawn from Modern Portfolio Theory (MPT) to create more efficiently diversified portfolios and, in this way, improve overall performance at a particular level of risk.2 Since the mid-1980's, real estate researchers and pension fund consultants have been describing the ideal or optimal real estate portfolio as one composed of mathematically-derived fractions of office, retail, industrial, and apartment properties. If another level of detail was desired, the researchers and

1 The most widely cited source for performance of investment-grade commercial real estate is the Russell-NCREIF Property Index. This industrywide benchmark has existed since December 1977 and, as of December 1991, consists of performance results for 1,622 unleveraged properties with an aggregate market value of approximately $22 billion.

consultants obliged with additional divisions of the ideal portfolio categorized across geographic or economic regions as well.\(^3\)

It is time to reconsider whether the techniques of MPT so common in portfolio considerations related to stocks and bonds are indeed valid for the real estate asset class and, more particularly, for decision-making within the real estate asset class. MPT has theoretical roots in the analysis of risky assets. In 1952, Harry Markowitz observed that risky assets could be described in terms of their average returns, the variance of those returns, and the covariance of each asset’s returns with every other risky asset’s returns.\(^4\) The data requirements to implement Markowitz’s model are too great to be practical, but by the early 1960’s, Sharpe, Lintner, and others developed simplified versions of MPT for common stocks.\(^5\) Later, these models were extended to bonds. Fortunately, both stocks and bonds exhibit performance characteristics that even the most critical observers admit fit the constraints of the MPT framework well enough to produce useful decisions.\(^6\) Useful decisions include determining allocations of capital between stocks and bonds as well as more refined allocations within the stock and bond asset classes themselves. Thus, most pension plans and their consultants today employ the mean-variance form of MPT in establishing asset allocations between and within asset classes.

Real estate has invariably been in a lagging position vis-à-vis stocks and bonds when it comes to both theoretical and practical concepts. Internal rate of return (IRR), for example, was widely known and utilized in the 1950’s in finance, but the real estate industry discovered it more than twenty years later.\(^7\)

Under pressure to provide a theoretical foundation for understanding both real estate performance and the reasons why real estate should be included in mixed-asset portfolios, real estate researchers and practitioners were quick to latch on to MPT. Early expositions of MPT as it might apply to real estate were made in the 1970’s, but the 1980’s created a groundswell of support for the concept. Unfortunately, lost in the rush to embrace MPT for real estate was any critical


\(^4\) Harry Markowitz, “Portfolio Selection,” Journal of Finance 7 (March 1952), pp. 77-91.


\(^6\) In its simplest construct, MPT is a technique for reduction of portfolio variance. This task is accomplished by constructing efficiently diversified portfolios at a level of risk, measured by the variance or standard deviation of returns, acceptable to the investor. The return on a portfolio of assets is the weighted average of the returns of the individual assets. While not strictly required, it is desirable that returns of individual assets be normally distributed. Weighted sums of normal variables are themselves normally distributed; that is, the normal distribution is stable in the sense that it reproduces itself under weighted addition. This additive property is important for the proper application of MPT, so it is useful to know the form of the distribution of returns of assets, like real estate, that might be included in a mixed-asset portfolio. Later, we will discuss ways in which the results from our sample portfolio differ from the normality condition.

analysis or understanding of the premises underlying the concept or its limitations in everyday use. Although no analytical concept is universally applicable, it is certainly helpful to have a rigorous theory and methodology to answer the myriad questions that investors pose concerning the efficient allocation of their capital. At the same time, anyone using an approach like MPT should be aware of its limitations.

**Some Necessary Conditions for Using MPT in Real Estate**

Application of MPT to real estate investment requires that individual properties in the same subclass behave roughly the same way. For example, if the analyst defines subclasses on the basis of property type or location, all office properties or all office properties in the same market must exhibit similar performance patterns for there to be a meaningful distinction between office properties and say retail properties. Therefore, within each subclass, real estate becomes essentially a commodity with near identical characteristics. Furthermore, it is important for diversifying a portfolio that properties in different subclasses have distinctly different patterns of returns. According to financial theory, volatility of returns, or risk, in the portfolio is reduced by mixing assets with different or offsetting performance patterns; that is, combining inherently risky assets that have low or negative correlations with one another reduces portfolio return volatility. 8

**Examining the Linkage Between Market Forces and Real Estate Performance**

Traditional thinking about real estate assumes that the investment performance of individual properties within a market moves in lock step with changes in overall market conditions. Strong local economies are thought to support strong real estate markets which in turn support strong property performance. In this way, investment performance is assumed to be linked closely to overall market conditions. One reason that analysts focus on market conditions rather than property performance is that considerable macroeconomic data are readily available for local or regional markets and no reliable source of data exists for property performance in a metropolitan area or other localized area. Consequently, researchers are forced to infer property performance indirectly.

Although the belief in a link between market performance and property performance is widespread among real estate researchers, this conclusion has not (with a few exceptions) been supported in the academic literature. Researchers have assumed that such factors as employment growth, vacancy rates, and capital availability affect property performance either positively or negatively, but they have not discovered the specific mechanism by which these factors produce property performance. 9

Indeed, they have undertaken little testing of these traditional beliefs for a variety of practical reasons. First, until recently data have not been collected on individual properties. Most of the performance data that does exist is proprietary and closely-guarded by investment managers and other property owners. Real estate is essentially a private market with privileged information,

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Unlike the stock and bond markets in which opinions of value and a wide array of learned opinions are available to the investor at relatively low cost. Finally, most available real estate performance data is aggregate data reported for entire classes of property, various geographic regions, or entire pools of real estate assets, rather than for individual properties at particular locations.

**Actual Performance of Commercial Property**

For two years, the authors have examined the performance of individual properties in the RREEF Funds portfolio, looking for patterns of returns. The portfolio includes roughly 200 industrial, retail, and office properties, some of which have return series extending back into the late 1970’s. All were unlevered equity investments purchased on behalf of public and corporate pension plans. Performance was measured using total annual time-weighted returns that combine income and appreciation. The discussion that follows focuses on a sample portfolio consisting of 50 properties for which there are complete annual performance figures for the years 1982 to 1991. The sample portfolio includes 23 industrial properties, 14 office properties, and 13 retail properties.

Both the sample data and the larger universe of data revealed that actual performance of individual property investments over time was volatile and relatively uncorrelated with that of similar properties, even in the same submarket. This finding is illustrated in Exhibit 1, which compares total returns of two 100,000-square-foot warehouse buildings located across the street from each other in the Kent Valley submarket of Seattle. Returns varied by as much as 20% from year to year, despite the much-discussed effect of appraisal smoothing. The patterns of returns were largely independent of one another (i.e., uncorrelated). In most years during the 1983 to 1991 period, the returns for one building moved up while those of the other moved down.

What was going on here? How could these two virtually identical properties, purchased at about the same time for nearly the same amount, owned and operated by the same company, exhibit such dramatically different performance?

**Individual Properties Matter**

The answer should not surprise anyone who has owned and managed real estate. The differences in returns exhibited by these two buildings were due to physical, operating, and financial characteristics of the individual properties themselves. Building A has a single tenant, while

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10 Annual total time-weighted returns are computed by chain-linking quarterly time-weighted returns in the manner used in the Russell-NCREIF Property Index. The formula for quarterly time-weighted returns is:

$$\text{Total Return} = \frac{\text{EMV} - \text{BMV} + \text{PS} - \text{CI} + \text{NI}}{\text{BMV} - 0.5 \text{PS} + 0.5 \text{CI} - 0.33 \text{NI}}$$

where \(\text{EMV}\) is the ending market value for the quarter, \(\text{BMV}\) is the beginning market value for the quarter, \(\text{PS}\) is partial sales proceeds, \(\text{CI}\) is capital improvements made in the quarter, and \(\text{NI}\) is net property income in the quarter. Partial sales and capital improvements are assumed to occur in mid-quarter, while net income is assumed to be received monthly. These presumptions account for the coefficients on the variables in the denominator.

11 David M. Geltner, “Bias in Appraisal-Based Returns,” American Real Estate and Urban Economics Association Journal 17 (1989), pp. 338-52; Stephen A. Ross and Randall C. Zisler, “Risk and Return in Real Estate,” Journal of Real Estate Finance and Economics 4 (June 1991), pp. 175-90; David M. Geltner, “Smoothing in Appraisal-Based Returns,” Journal of Real Estate Finance and Economics 4 (September 1991), pp. 327-45. It should be noted that the “problem” of appraisal bias is most relevant when comparisons are made between real estate, an infrequently-traded asset, and stocks or bonds, actively-traded financial assets. So long as we confine our investigation to the real estate asset class itself, the valuation issue is of limited importance.
Building B has two. Leases expired at different times for the different tenants and were rolled over at different prevailing market rents. A roof was replaced on one building. Myriad similar property-specific factors sent returns in different directions, even though both buildings are functionally and locationally similar and, naturally, subject to the same overall market conditions. This does not mean that market conditions were not important or that they did not have an influence on investment performance. They did. It means that there were other, probably more influential, property-specific factors at play between the market and the bottom line that produced quite different patterns of returns.

In other words, performance of the market is not tantamount to performance of the property. This reinforces the observation made earlier that to date there is no proven mechanism by which markets and properties are linked together. Property performance is specific to the property not generic to the property type or location.

Stated another way, property-specific circumstances have a dominant influence on performance of individual properties. External market forces affect a property's performance at differential rates. For example, some properties may respond quickly to changes in market rent, while others may be slow to respond. Clearly, the time until lease rollover and the relation between current and market rents are two of the factors that might influence the responsiveness to external market forces.12

The same volatility and relative independence of individual property returns appeared throughout the sample portfolio, regardless of how properties were clustered. Exhibit 2 compares individual industrial property returns in Los Angeles, Chicago, Dallas, and Houston. Exhibit 3 compares returns for different property types in Los Angeles during the same time period. These illustrations tell a powerful story: individual property returns are quite volatile and independent of one another.

Effect of Grouping Properties into Portfolios

Grouping properties together produces an immediate portfolio effect, or reduction in the overall volatility of returns of the group, because of the statistical independence of individual property returns. The ups and downs in property-level returns tend to offset one another when properties are grouped together such that the performance of a portfolio of properties is significantly less volatile than the performance of its individual property components. This offsetting effect appears to be more significant than appraisal smoothing as a cause of the much criticized low volatility in NCREIF Indices and in fund-level returns.

Testing Property Return Patterns Over Time

Another way to search for similarities in returns among similar properties or among similar properties in the same market is to calculate correlation coefficients between each possible pair of properties. Properties with patterns of returns that move in exactly the same way will produce a

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coefficient of +1. Those that move opposite to one another are said to be negatively correlated and given a coefficient of -1. Returns that have no discernable relationship to one another are called uncorrelated and given a coefficient of 0.

A large portfolio that mixed different types of property from different locations would be expected to produce a wide range of correlation coefficients with an average value of close to 0. This would mean that, on average, the individual returns were uncorrelated, thereby reducing the overall return volatility of the portfolio. If graphed to show the frequency distribution of the coefficients, a pattern like that shown in the uppermost graph of Exhibit 4 would be expected. This is a Gaussian normal distribution, with the largest concentration centered over 0.13 This is roughly what we found with the sample portfolio, but the shape of the distribution depends upon the period chosen. For example, the center graph of Exhibit 4 covering the period 1982 to 1990 appears to conform reasonably well to the expectations. It has a mean of 0.10, a slight negative skewing (the longest tail of the distribution is on the negative side of the mean), and a somewhat flattened distribution relative to the Gaussian normal distribution. The bottom graph of Exhibit 4 shows results for the same 50 properties covering the period 1982 to 1991, an advance of only one year. This graph has a higher mean of 0.24, a greater negative skewing, and about the same amount of flattening relative to the Gaussian normal distribution. The reason for the differences between these two graphs is to be found in the sharp downward valuations that occurred in 1991 in response to a general market revaluation. The downward valuations affected virtually all properties in the sample portfolio; thus, the mean of the correlation coefficients became much more positive.

If properties of the same type or in the same market are truly a commodity in the sense that they behave the same way, the mean of their correlations would be positive and there would be considerable negative skewing of the distribution like that shown in the top graph of Exhibits 5A and 5B. This, however, is not the distribution of correlation coefficients that emerged from comparison of returns from properties of the same type: industrial, office, and retail. The results are shown on the three other graphs of Exhibits 5A and 5B for the periods 1982 to 1990 and 1982 to 1991, respectively. The distributions are not skewed to the right of the 0 point and, in fact, the average of the correlation coefficients is very close to 0 as shown in Exhibit 6. This indicates that the returns for properties of the same type are also relatively independent. The portfolio effect is attainable with properties of the same type. In other words, it does not appear necessary to have a mixed property-type portfolio to lower the volatility of returns for the overall portfolio; lower portfolio return volatility can be achieved even with a combination of properties of identical type.

Exhibit 6 shows a summary of descriptive statistics for the distribution of correlation coefficients for combinations of the 50 properties of the sample portfolio over two periods: 1982 to 1990 and 1982 to 1991. In addition to analyzing the coefficients for combinations of the same type of property (e.g., industrial versus industrial, office versus office, and retail versus retail), this exhibit shows results for three other possible combinations: industrial versus office, industrial versus retail, and office versus retail. The analysis compared the coefficients for combinations of

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13 It should be noted that distributions of correlation coefficients cannot be Gaussian normal distributions because correlations are bounded by the numbers -1 and +1. An appropriate way to analyze a distribution of correlation coefficients would be to apply the Fisher z-transformation to the distribution of correlation coefficients, but that technique is beyond the scope of this article. Thus, the references to normal distributions and statistics derived from the distributions that follow should be regarded as approximations rather than mathematically rigorous results.
properties within and across type to investigate whether a particular set deviated significantly from all the others. For both periods, the following generalizations are possible:

- The mean of any selected set of correlation coefficients is not statistically different than the mean of all correlation coefficients. For example, the mean of the industrial-versus-industrial set is not statistically different from the mean of all correlation coefficients.
- The most statistically significant results for one period change dramatically when the period is altered, leading to the observation that the results are critically dependent upon the period covered. For example (focusing on the office-versus-office set), in the 1982 to 1990 period, the probability that this mean would differ from the mean of all correlation coefficients is about 68%, but advancing the analysis one more year to cover the period 1982 to 1991 reduces that probability to about 46%.
- The standard deviations of the distributions of correlation coefficients are remarkably similar. The range of the standard deviations are 0.392 to 0.447 for the period 1982 to 1990 and 0.374 to 0.422 for the period 1982 to 1991. This suggests that the individual property returns when viewed over time are, in general, considerably different from one another; that is, the risk of real estate is largely nonsystematic or property-specific risk, not market-driven or systematic risk.
- The skewness of each distribution is generally negative. It appears that the distribution of correlation coefficients for real estate returns is more accurately described as log-normal rather than normal. Also, although the kurtosis of a normal distribution is 3.0, virtually all of the distributions analyzed including many not shown in this article, have a kurtosis of between 1.8 and 2.2, meaning that the distribution is somewhat flatter than the normal distribution; (i.e., platykurtotic). These two observations cast additional doubt upon the wisdom of using MPT on an asset class that deviates so greatly from the Gaussian normal distribution essential to proper use of the theory.

Unfortunately, the sample of properties in this study was not large enough to run the various statistically reliable tests for similar properties in the same market. Nonetheless, a look at the patterns of returns of actual properties shown in Exhibits 1, 2, and 3 suggests that the results would be similar. Again, the property-level differences among properties are the most significant determinants of total returns, not the property type or location. The due to similarities among properties must be found in other descriptors.

Is Real Estate Different from Stocks and Bonds?

This study suggests that the performance of equity real estate differs from that of stocks and bonds. Individual real estate investments appear to be more independent of one another than individual stocks or bonds. This difference (illustrated conceptually in Exhibit 7) results in a distinctly different performance pattern for the real estate asset class.

Bonds prices (and yields) are highly correlated with one another, and the range of correlation coefficients is rather small. This correlation is due to the fact that bond prices are inextricably linked to movements in interest rates. Stocks are less highly correlated simply because any particular stock price is tied to the company's unique earnings prospects. Nonetheless, that stocks as a whole are positively correlated can be seen by the significant correlation of the various stock
market indices, regardless of the differences in how those indices are constructed or the types or market capitalization of the stocks included. A analysis of equity real estate produces a distinctly different result. Although the mean of correlations may have a positive value for some periods, the results are both mixed and less positive than those of either stocks or bonds. This fact alone makes a strong case for treating real estate as a distinct asset class with unique behavioral characteristics.

The results also make a powerful argument for including real estate in a mixed-asset portfolio as a diversifier, if not as a superior return generator. Because real estate has relatively low correlation with assets like stocks and bonds, it can damp the portfolio's return volatility without imposing the severe penalty of lower overall portfolio return.14 Ironically, securitizing equity real estate as is now being widely discussed will tend to make the performance expectations of real estate more like those of other securities, so that the current apparent differences between real estate and other asset classes may be muted in a future, securitized world.

Conclusions

Although this article may appear to be an indictment of Modern Portfolio Theory and its applicability to real estate investment, that is neither its intent nor its conclusion. MPT has stood the test of time as a guide to broad asset allocation decision-making across entire asset classes: stocks, bonds, and real estate. There is ample evidence that MPT performs reasonably well within the stock and bond asset classes that have a long history of factual data to support the theoretical implications. However, the analysis here shows that MPT has little applicability within the real estate asset class itself, at least as a guide to distinctions among real estate choices on the basis of physical or locational attributes.

Analysis of the performance of actual commercial properties leads to the following conclusions:

- Investment returns are volatile at the individual property level primarily because of the particular circumstances of the properties.
- Total returns are relatively independent of property type or location.
- To the extent that patterns of total returns exist at all, they do so for relatively short periods.
- Patterns of total returns do not appear to be characteristic of an individual property type or of properties within geographic or economic regions.
- Reduced volatility of returns for portfolios of real estate is due more to independence of individual property returns and the low correlation among property returns than to appraisal smoothing.

Thus, it appears that too much emphasis is being placed on constructing portfolios across property types and locations. The two factors by themselves are not sufficient to determine optimal portfolio balance or superior performance at a given level of risk. Furthermore, to follow the dictates of MPT for asset allocation decisions within the real estate class would undoubtedly trigger frequent and costly rebalancing of the portfolio.

Analysts must investigate property-level factors that influence investment performance before they uncritically apply, within the asset class, allocation techniques borrowed from MPT. Such factors as tenant credit quality, lease duration, contract-to-market rent disparity, and costs of rollover have a considerable impact on reported performance and react differently to external factors.

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market forces. More work needs to be done to isolate those factors that truly distinguish one property from another in an unambiguous way.

Most importantly, it appears that such analysis can lead to substantial reduction in portfolio-level return volatility within a given property type or within a metropolitan area, a reduction of the same order of magnitude as is achieved by spreading investments widely across the country or among various property types. Thus, before investors make allocation decisions, they should weigh the potential benefits of spreading investments widely against the potential costs of such diversification in terms of investment quality, effective control, management expertise and efficiency, and potential net returns.
Exhibit 1
Investment Performance of Similar Warehouse Properties
Kent Valley Submarket, Seattle, WA

Property A
Property B
Exhibit 2
Industrial Property Returns by Metropolitan Area

Total Annual Return

Los Angeles
Chicago
Dallas
Houston

82 83 84 85 86 87 88 89 90 91
Exhibit 3
Returns in Los Angeles County by Property Type

- Industrial
- Office
- Retail
Exhibit 4
Distribution of Correlation Coefficients
Traditional Expectations and Actual Results

Industrial, Office, and Retail Properties
Correlations of Annual Total Returns 1982 to 1990

<table>
<thead>
<tr>
<th>property type</th>
<th>no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>23</td>
</tr>
<tr>
<td>Office</td>
<td>14</td>
</tr>
<tr>
<td>Retail</td>
<td>13</td>
</tr>
</tbody>
</table>

Industrial, Office, and Retail Properties
Correlations of Annual Total Returns 1982 to 1991

<table>
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<th>no.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>23</td>
</tr>
<tr>
<td>Office</td>
<td>14</td>
</tr>
<tr>
<td>Retail</td>
<td>13</td>
</tr>
</tbody>
</table>
Exhibit 5A
Distribution of Correlation Coefficients
Traditional Expectations and Actual Results
Industrial, Office, and Retail Property Annual Returns 1982 to 1990

Traditional Expectations

-1 +1

Industrial Properties

Office Properties

Retail Properties
Exhibit 5B
Distribution of Correlation Coefficients
Traditional Expectations and Actual Results
Industrial, Office, and Retail Property Annual Returns 1982 to 1991

- Industrial Properties
- Office Properties
- Retail Properties
### Exhibit 6

Statistical Analysis of Correlation Coefficients for 50 Industrial, Office, and Retail Properties

#### For the Period 1982 to 1990:

<table>
<thead>
<tr>
<th></th>
<th>Industrial vs. Industrial</th>
<th>Office vs. Office</th>
<th>Retail vs. Retail</th>
<th>Industrial vs. Office</th>
<th>Industrial vs. Retail</th>
<th>Office vs. Retail</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.052</td>
<td>0.254</td>
<td>0.113</td>
<td>0.068</td>
<td>0.112</td>
<td>0.117</td>
<td>0.099</td>
</tr>
<tr>
<td><strong>Std Deviation</strong></td>
<td>0.414</td>
<td>0.392</td>
<td>0.418</td>
<td>0.447</td>
<td>0.394</td>
<td>0.428</td>
<td>0.421</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.22</td>
<td>0.13</td>
<td>-0.16</td>
<td>-0.01</td>
<td>-0.25</td>
<td>-0.25</td>
<td>-0.15</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>2.14</td>
<td>1.82</td>
<td>2.19</td>
<td>1.98</td>
<td>2.33</td>
<td>2.14</td>
<td>2.18</td>
</tr>
<tr>
<td><strong>t-stat (H₀)</strong></td>
<td>-0.277</td>
<td>1.010</td>
<td>0.076</td>
<td>-0.157</td>
<td>-0.082</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td><strong>P(H₀ is true)</strong></td>
<td>0.778</td>
<td>0.316</td>
<td>0.938</td>
<td>0.870</td>
<td>0.932</td>
<td>0.922</td>
<td></td>
</tr>
</tbody>
</table>

#### For the Period 1982 to 1991:

<table>
<thead>
<tr>
<th></th>
<th>Industrial vs. Industrial</th>
<th>Office vs. Office</th>
<th>Retail vs. Retail</th>
<th>Industrial vs. Office</th>
<th>Industrial vs. Retail</th>
<th>Office vs. Retail</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.198</td>
<td>0.331</td>
<td>0.251</td>
<td>0.216</td>
<td>0.256</td>
<td>0.251</td>
<td>0.238</td>
</tr>
<tr>
<td><strong>Std Deviation</strong></td>
<td>0.409</td>
<td>0.385</td>
<td>0.374</td>
<td>0.422</td>
<td>0.380</td>
<td>0.410</td>
<td>0.403</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.41</td>
<td>0.04</td>
<td>-0.25</td>
<td>-0.19</td>
<td>-0.47</td>
<td>-0.40</td>
<td>-0.33</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>2.26</td>
<td>1.60</td>
<td>2.14</td>
<td>1.77</td>
<td>2.51</td>
<td>2.18</td>
<td>2.16</td>
</tr>
<tr>
<td><strong>t-stat (H₀)</strong></td>
<td>-0.239</td>
<td>0.625</td>
<td>0.093</td>
<td>-0.123</td>
<td>0.125</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td><strong>P(H₀ is true)</strong></td>
<td>0.806</td>
<td>0.541</td>
<td>0.924</td>
<td>0.898</td>
<td>0.896</td>
<td>0.937</td>
<td></td>
</tr>
</tbody>
</table>

The null hypothesis is that the true mean of the distribution of a set of correlation coefficients is equal to the mean of the distribution of all correlation coefficients. No result was statistically significant at the 1%, 5%, or 10% levels. Thus, the null hypothesis cannot be rejected.
Exhibit 7
Conceptual Frequency Distribution of Correlation Coefficients
Real Estate versus Stocks versus Bonds

Real Estate

Stocks

Bonds
The efficient frontier can also be calculated formulaically, but simulating and visualizing portfolio runs can help better illustrate what's going on. If the above is all sounding like a lot, don't worry, it will become clearer as I step through the code example below.

2) Setting up a random portfolio of equities. First, let's grab some historical return data for a set of equities. We can then use this to generate random portfolios in our simulation.

```python
import pandas as pd
from pandas_datareader import data as pdr
```

Along the Efficient Frontier. Investment performance of individual properties depends less on property type and location than generally believed. Regarded as approximations rather than mathematically rigorous results. Along The Efficient Frontier properties within and across type to investigate whether a particular set deviated significantly from all the others. The Efficient Frontier is a curve that follows the highest return at every risk level. In the picture below, the Efficient Frontier is the green curve. According to MPT, no investor should ever be invested in a portfolio other than the ones found along this curve. Note: an interesting effect would take place if we included a risk-free asset (i.e., cash) in our portfolio mix. Our curve would actually become a straight line and would exhibit a different shape altogether.

Part III. Python Implementation. Let's now go over some of the Python code used in this implementation and try to generate some